cope with the upwash $W_j(x,t)$ given by

$$W_j(x,t) = V\frac{\partial Z_j}{\partial x}(x,t) + \frac{\partial Z_j}{\partial t}(x,t)$$

$$= V\left\{c\frac{\mathrm{d}}{\mathrm{d}x}f_j(x)q_j(t) + \frac{c}{V}f_j(x)\frac{\mathrm{d}q_j(t)}{\mathrm{d}t}\right\} \tag{22}$$

The time interval T must be so chosen that $T<2n\pi/\Omega$ and the time step for the finite difference approximation be chosen correspondingly to be small enough for adequate accuracy to be obtained. The number of time steps required before the generalized air force $K_{jk}(t)$ is effectively zero will have to be determined by experience, but it will depend on n, T, and the time step.

IV. Conclusions

A suitable form of the generalized coordinate $q_j(t)$, defining the modal motion of an airfoil, has been prescribed as a transient in which there are no changes severe enough to mar the accuracy of finite difference approximations to governing aerodynamic equations. The form of $q_j(t)$ is an approximation to the Dirac delta function. From the resulting generalized air forces $K_{jk}(t)$, it is possible to determine the harmonic air force coefficients $\bar{Q}_{jk}(\omega)$ that are required for aeroelastic analysis.

References

[1] Ballhaus, W. F. and Goorjian, P. M., "Computation of Unsteady Transonic Flows by the Indicial Method," *AIAA Journal*, Vol. 16, Feb. 1978, pp. 117-124.

[2] Rizzetta, D. P. and Chin, W. C., "Effect of Frequency in Unsteady Transonic Flow," *AIAA Journal*, Vol. 17, July 1979, pp. 779-781.

Asymptotic Features of Shock-Wave Boundary-Layer Interaction

M. Y. Hussaini,* B. S. Baldwin,† and R. W. MacCormack‡
NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

C = Chapman-Rubesin constant, $[(T_0+198.6)/(T_w+198.6)](T_w/T_0)^{1/2}$
C_f = $[\mu(\partial u/\partial y)]_{y=0}/\tfrac{1}{2}\rho_0 U_0^2$
M_0 = freestream Mach number
p = pressure (dimensional)
p_b = minimum surface pressure that occurs immediately ahead of interaction region
p_0 = freestream pressure (undisturbed)
P = $(M_0^2-1)^{1/4}[p(x_s)-p_0]/(R^{1/4}\gamma C^{1/4}M_0^2 p_0)$
P' = $[R^{1/8}(T_w/T_0)^{3/2}C^{1/8}]/[\gamma M_0^2(M_0^2-1)^{1/8}\lambda^{7/4}](\mathrm{d}p/\mathrm{d}x)|_{x=x_s}$
R = Reynolds number, $U_0 x_s/\nu_0$
T_w = temperature at wall
T_0 = temperature in undisturbed freestream
U_0 = undisturbed freestream velocity
U_1 = velocity behind incident shock
x_s = distance of separation point from leading edge
X = $[(x-x_s)/x_s](K/C)^{3/8}(M_0-1)^{3/8}\lambda^{5/4}(T_0/T_w)^{3/2}$
λ = 0.3321

Received Oct. 15, 1979; revision received Jan. 28, 1980.

Index categories: Shock Waves and Detonations; Jets, Wakes, and Viscid-Inviscid Flow Interactions; Supersonic and Hypersonic Flow.

*Research Scientist, presently at ICASE, Langley Research Center, Hampton, Va.

†Research Scientist, Computational Fluid Dynamics Branch. Member AIAA.

‡Staff Scientist, Thermo- and Gas/Dynamics Division. Member AIAA.

Introduction

IT is well known that when a sufficiently strong shock wave strikes a laminar boundary layer, the boundary layer separates ahead of the point of shock incidence, and the flow features of the separation region are independent of the mode of inducing separation and the direct influences of downstream geometry. Chapman et al.[1] were the first to give a simple physical explanation of this free-interaction phenomenon. A self-consistent mathematical theory was developed by Lighthill[2] for interaction of a weak shock wave with a boundary layer. The generalization of this linear theory to include nonlinear disturbances on the boundary-layer scale led to the triple-deck formulation of Stewartson and Williams[3] and Neiland.[4] A comprehensive review of this asymptotic large Reynolds number theory is given by Stewartson.[5] Since solutions based on the triple-deck theory can be reliably evaluated, it is of interest to determine the Reynolds number range within which they are accurate so that they may be used to check the accuracy of more complex solution procedures.

The purpose of the present Note is to apply the semi-implicit method of MacCormack[6] to solve the Navier-Stokes equations numerically and to evaluate certain features of the free-interaction phenomenon that occurs when a shock wave impinges on a Blasius boundary layer. Comparisons are made with predictions of the triple-deck theory and experiment. The results include pressure and skin-friction distributions in the free-interaction region for various values of Reynolds number. The functional dependence on Reynolds number of the shock strength required for incipient separation is determined. The triple-deck (asymptotic) theory is viewed in the light of these results, and conclusions are drawn with regard to this theory being used as a test for numerical schemes.

Results and Discussion

A. Incipient Separation

Incipient separation is that condition in which the wall shear is positive everywhere except at one point where it vanishes. Estimates of the scaling of the magnitude of the required shock strength and the streamwise extent of its interaction with the boundary layer have been given by Neiland[4] and Sychev.[7] According to these estimates, the pressure scales as $R^{-1/4}$ and the streamwise extent of interaction scales as $R^{-3/8}$.

To determine incipient separation by computation for a given Mach number and Reynolds number, two angles of shock incidence are guessed, such that for one of them the flow remains attached and for the other it separates. The interval between these two angles is divided into sub-intervals of, for example, 0.1, and the flowfield is computed at these shock incidences. The results then give two relatively close angles. The boundary layer separates for one angle, but not for the other. The interval between these two angles can be further subdivided and so on until the desired accuracy is obtained. The numerical results obtained bear out the preceding estimate of dependence on Reynolds number. In Fig. 1, the scaled shock strength for incipient separation determined from the present calculations is plotted vs Reynolds number. Straight horizontal lines have been drawn through the computed points. The deviation of the computed points from the straight line predicted by asymptotic theory

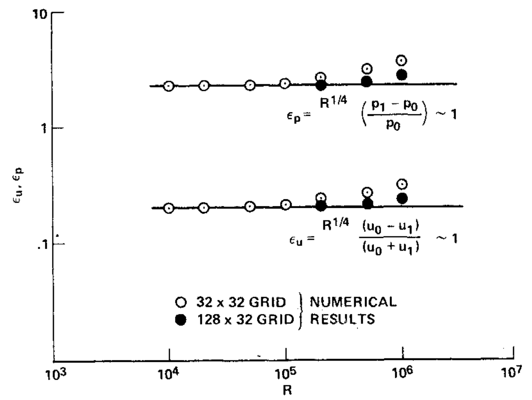

Fig. 1 Shock strength for incipient separation vs Reynolds number.

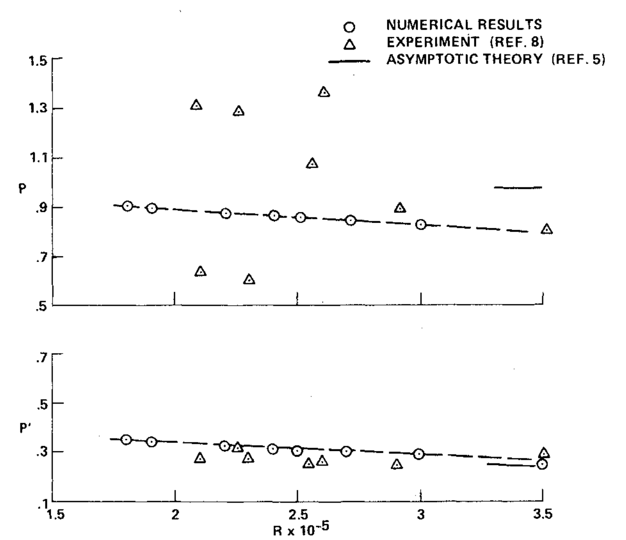

Fig. 2 Comparison of numerical, experimental, and asymptotic values of scaled pressure and pressure gradient at the separation point.

may be interpreted to mean that the numerical results obtained on a 32×32 grid become rather inaccurate for Reynolds number beyond 2×10^5. The reason appears to be that the relevant streamwise length scale of the interaction region has not been resolved. [The exponentially stretched fine-mesh region of height proportional to $4(T_w/T_0)^{0.7}/\sqrt{R}$, employed adjacent to the wall in the present calculations, has resolved the smallest-length scale suggested by the asymptotic theory in the direction normal to the wall.] When the number of nodal points in th streamwise direction is increased by a factor of 4 (128×32 mesh), the agreement with theory is found to extend to a Reynolds number of about 10^6. This implies that, given a grid and a method of certain order of accuracy, there is an upper limit on the Reynolds number for which the flowfield can be simulated numerically.

B. Shock-Induced Separated Flow

According to the triple-deck theory, the free-interaction layer centered on the separation point has a streamwise length scale $0(R^{-3/8})$ and three sub-boundary layers called decks. In the main deck, whose thickness is $0(R^{-1/2})$, the solution is an inviscid perturbation of the Blasius profile at the separation point. This is matched with the outer potential flow through the simple-wave solution of the upper deck with thickness $0(R^{-3/8})$. The velocity of slip of the main-deck solution is reduced to zero in the lower deck of thickness $0(R^{-5/8})$, where the incompressible boundary-layer equations hold with novel boundary conditions permitting interaction with the overlying layers. Three types of solutions seem to exist for these equations; the one relevant to the present case is said to have

Table 1 **Characteristic properties of p in free interactions, $M_0 = 2.4$**

Scaled pressure	Source	$R = 0.02 \times 10^6$	$R = 0.1 \times 10^6$
	Chapman et al.[1]	0.88	0.68
P	Present results	1.25	1.21
	Asymptotic theory	0.86	0.86
	Chapman et al.[1]	0.45	0.38
P'	Present results	0.44	0.39
	Asymptotic theory	0.59	0.53

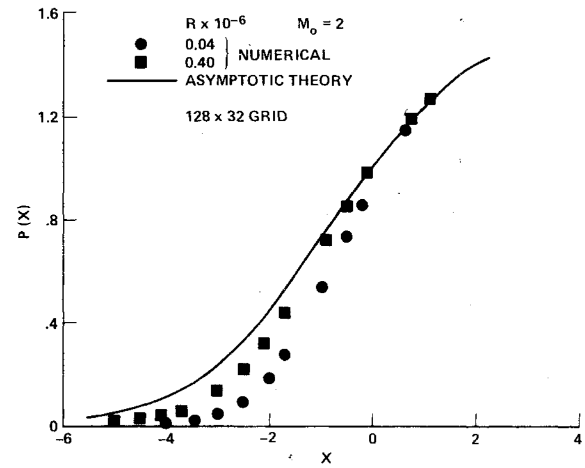

Fig. 3 Surface-pressure distribution in the free-interaction region.

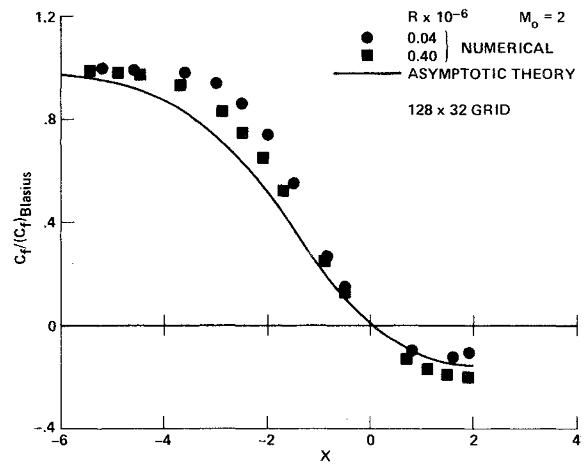

Fig. 4 Skin-friction distribution in the free-interaction region.

been initiated by increasing pressure slightly from its upstream static value, whereupon the solution evolved automatically until separation occurred.[3] This theory does not take into account an additional condition on velocity at the end of streamwise integration, and hence, its results may be doubtful in part of the reversed-flow region. Perhaps it should be mentioned that an attempt to include second-order terms in the triple-deck model by Ragab and Nayfeh[8] has led to a generalized method of multiple scales. However, no calculations by this method have been carried out as yet for the present problem.

In Fig. 2, we compare the values of the scaled pressure P (see nomenclature) and its gradient P' at the point of separation (zero skin friction) obtained by the present numerical method[6] on a 32×32 grid, the asymptotic theory, and experiment as quoted by Stewartson.[5] (The experimental values have been measured from the graphs of Hakkinen et al.[9]) The experimental data for P show wide scatter. For P', there is better agreement between numerical, experimental, and asymptotic results. In Table 1, calculated values of P and P' are compared with the data of Chapman et al.[1] Again, it is noted that, while the results for P' agree very well, those for P do not. This may be due to experimental inaccuracies in the location of the separation point magnified by scaling factors. A more disconcerting discrepancy is that as the Reynolds number increases, the numerical value of pressure at separation moves away from the asymptotic value. As in the case of incipient separation, the discrepancy between numerical and asymptotic results is due to the fact that the 32×32 grid does not resolve the interaction region sufficiently. To improve resolution, the computations were carried out on a 128×32 grid with other parameters unchanged. Figures 3 and 4 present the pressure and skin-friction distributions in the free-interaction region for Reynolds number equal to 0.04×10^6 and 0.4×10^6. Results for the intermediate values of Reynolds number fall consistently between those shown. The scaled pressure P at separation ($x = 0$) now approaches the asymptotic value as the Reynolds number increases. It should be noted that in the asymptotic analysis, p_0 is the surface pressure in the Blasius case and is equal to the freestream pressure. In the numerical computations for finite Reynolds number, the surface pressure p_b is different from p_0 due to the effect of boundary-layer displacement thickness. In Fig. 3, the scaled pressure P is referred to p_b instead of p_0, and hence, P tends to zero ahead of the interaction region, whereas the scaled pressure based on p_0 would not tend to zero.

Conclusions

For the range of physical and numerical parameters considered, numerical methods of the type employed here are accurate enough for practical purposes. Moreover, it can generally be concluded that the asymptotic theory[5] is useful as a test bed for numerical schemes developed for computation of complex flow configurations for design purposes. The mesh resolution suggested by the asymptotic scaling laws are of value in numerical simulation codes which are expected to yield reliable results difficult to obtain in the laboratory. Application of a numerical method to simple problems, which may not necessarily be of practical importance but which possess precise solutions, can help in evaluating the capabilities and limitations of the method.

Acknowledgment

The first author was partially supported under NASA Contracts NAS1-14101 and NAS1-14472 while in residence at ICASE, NASA Langley Research Center, Hampton, Va.

References

[1] Chapman, D. R., Kuehn, D. M., and Larson, H. K., "Investigation of Separated Flows in Supersonic and Subsonic Streams with Emphasis on the Effects of Transition," NACA Rept. 1356, 1958.

[2] Lighthill, M. J., "On Boundary Layers and Upstream Influence. A Comparison between Subsonic and Supersonic Flows," *Proceedings of the Royal Society, London*, Vol. A217, May 1953, pp. 478-507.

[3] Stewartson, K. and Williams, P. G., "Self-Induced Separation," *Proceedings of the Royal Society, London*, Vol. A312, Sept. 1969, pp. 181-206.

[4] Neiland, V. Y., "Towards a Theory of Separation of a Laminar Boundary Layer in Supersonic Stream," *Izvestia Akademii Nauk SSSR, Mekhanika Zhidkostii Gaza*, Vol. 4, No. 4, 1969 (*Fluid Dynamics*, Vol. 4, No. 4, July 1969, pp. 33-35).

[5] Stewartson, K., "Multistructured Boundary Layers on Flat Plates and Related Bodies," *Advances in Applied Mechanics*, Vol. 14, Academic Press, Inc., New York, 1974, pp. 145-239.

[6] MacCormack, R. W., "An Efficient Explicit-Implict-Characteristic Method for Solving the Compressible Navier-Stokes Equations," *SIAM-AMS Proceedings*, Vol. 11, 1978, pp. 130-155.

[7] Sychev, V. V., "Laminar Separation," *Izvestia Akademii Nauk SSSR, Mekhanika Zhidkostii Gaza*, Vol. 7, No. 3, May 1972, pp. 407-417.

[8] Ragab, S. A. and Nayfeh, A. H., A Second-Order Asymptotic Solution for the Laminar Separation of Supersonic Flows Past Compression Ramps," AIAA Paper 78-1132, Seattle, Wash., July 1978.

[9] Hakkinen, R. J., Trilling, G. L., and Abarbanel, S. S., "The Interaction of an Oblique Shock Wave with a Laminar Boundary Layer," NASA Memo 2-18-59W, March 1959.

Mach and Reynolds Number Effects on a Shock-Wave/Boundary-Layer Interaction

G. G. Mateer* and J. R. Viegas†
NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

c_f = skin friction coefficient
M = Mach number
p = pressure
Re = Reynolds number
x = axial coordinate
δ = boundary-layer thickness

Subscripts

s = distance along tunnel wall
∞ = freestream value just ahead of the shock wave

Introduction

THE present investigation was undertaken as part of a continuing experimental/numerical program to evaluate and improve turbulence models for use in Navier-Stokes (N-S) codes. The normal shock-wave/turbulent boundary-layer interaction is a good test for such computations because it contains strong adverse pressure gradients and the possibility of local flow separation.

Experimental studies of these interactions are usually conducted on wind tunnel walls or large flat plates that have thick, easily documented boundary layers. However, the constraining effect of the tunnel walls may prevent the flow from developing as it would in free air.[1] Since constraints must be included in any computational scheme, methods employing the N-S equations are attractive because simultaneous treatment of both the viscous and inviscid flowfields is possible. The evolution of N-S codes is based primarily upon the development of models for the turbulence terms in these equations. Earlier work[2] provided a data base at single Mach and Reynolds numbers which was used[3] to evaluate various types of eddy viscosity models of the turbulence (algebraic, one-, and two-equation, etc.). In the present study of the most promising model[4] is tested over a wider range of Mach and Reynolds numbers.

Presented as Paper 79-1502 at the AIAA 12th Fluid and Plasma Dynamics Conference, Williamsburg, Va., July 23-25, 1979; submitted Aug. 27, 1979; revision received Jan. 21, 1980. This paper is declared a work of the U.S. Government and therefore is in the public domain.

Index categories: Transonic Flow; Boundary Layers and Convective Heat Transfer—Turbulent; Computational Methods.

*Research Scientist.

†Research Scientist. Member AIAA.